INDIAN STATISTICAL INSTITUTE (BANGALORE)
SEMESTRAL EXAMINATION
II SEMESTER, 2009-2010
ANALYSIS IV
INSTRUCTOR: K. RAMA MURTHY

Max. marks: 100 Time Limit: 3hrs

1. Recall that a subset A of a metric space X is said to be totally bounded if for every $\epsilon > 0$ we can find a finite number of open balls of radius ϵ that cover A. Show that A is totally bounded if and only if its closure is totally bounded. [15]

2. a) Prove or disprove:
The vector space spanned by $1, \sin x, \sin(2x), ...$ is dense in $C[0,1]$. [10]

2. b) Find the closed subspace of $C[0,1]$ spanned by $\sin(x), \sin(2x), \sin(3x), ...$ [15]

3. Prove that $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin(nx)$ converges for all $x \in \mathbb{R}$. [25]

Hint: Let $a_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$ and write the series in terms of a_n's.

4. Prove that $\int\limits_{-\pi}^{\pi} \sin^3(x)\sin(203x)dx = 0$. [10]

5. Let $\sum\limits_{n=1}^{\infty} |a_n| < \infty$ and $\sum\limits_{n=1}^{\infty} |b_n| < \infty$. Let $f(x) = \sum\limits_{n=1}^{\infty} a_n e^{-inx}, g(x) = \sum\limits_{n=1}^{\infty} b_n e^{inx}$. Show without any computation that $f * g = 0$. [10]

6. Let f be a continuous function on $[-\pi, \pi]$. Show that $\int\limits_{-\pi}^{\pi} f(x)\sin(tx)dx \to 0$ as $t \to \infty$. [15]